Since $x=h$ when $t=0$, $C=\dfrac{4K\sqrt{h}}{ab\sqrt{2g}}$.

$$\therefore t=T=\frac{4K}{ab\sqrt{2g}}(\sqrt{h}-\sqrt{x}). \quad \therefore x=\left(\frac{4K\sqrt{h}-Tab\sqrt{2g}}{4K}\right)^2.$$

AVERAGE AND PROBABILITY.

98. Proposed by REV. PREBENDARY WHITWORTH, A. M.

A has £m and B has £n. They play for points until one of them has lost all his money. If α and β be the respective chances that A and B win any point, the expectation of the number of points played will be

$$\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n+\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let A_m=A's chance of winning, B_n=B's chance of winning.

Then nA_m, mB_n=A's and B's expectation, respectively.

$\therefore$ Expectation of number of points played$=E$.

Then $E=(nA_m-mB_n)/(\alpha-\beta)$.

Let A_x=A's chance when he has x pounds and B has $m+n-x$ pounds.

$$\therefore A_x=\frac{\beta}{\alpha+\beta}A_{x-1}+\frac{\alpha}{\alpha+\beta}A_{x+1}.$$

$A_x-A_{x-1}=(\alpha/\beta)(A_{x+1}-A_x)$. Giving x successive values from 1 to x we get $A_1-A_0=(\alpha/\beta)(A_2-A_1)$, $A_2-A_1=(\alpha/\beta)(A_3-A_2)$, etc.

By continued multiplication we get $A_1-A_0=(\alpha/\beta)^{x-1}(A_x-A_{x-1})$ or $A_x-A_{x-1}=(\beta/\alpha)^{x-1}(A_1-A_0)$.

Give x successive values from 1 to x and add

$$A_x-A_0=(A_1-A_0)[1+\beta/\alpha+(\beta/\alpha)^2+\ldots\ldots+(\beta/\alpha)^{x-1}].$$

But $A_0=0$. $\therefore A_x=A_1[1-(\beta/\alpha)^x]/[1-(\beta/\alpha)]$.

$A_{m+n}=1$. $\therefore 1=A_1[\alpha^{m+n}-\beta^{m+n}]/[\alpha^{m+n-1}(\alpha-\beta)]$.

$\therefore A_1=[\alpha^{m+n-1}(\alpha-\beta)]/(\alpha^{m+n}-\beta^{m+n})$.

$\therefore A_x=[\alpha^{m+n-1}(\alpha^x-\beta^x)]/[\alpha^{x-1}(\alpha^{m+n}-\beta^{m+n})]$.

$\therefore A_m=[\alpha^n(\alpha^m-\beta^m)]/(\alpha^{m+n}-\beta^{m+n})$.

Similarly, $B_n=[\beta^m(\alpha^n-\beta^n)]/(\alpha^{m+n}-\beta^{m+n})$.

$$\therefore E=\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n-\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

99. Proposed by E. B. SEITZ.

A point is taken at random in the surface of a given circle, and from it a line equal in length to the radius is drawn, so as to lie wholly in the surface of the circle. Find the chance that the line intersects in a given diameter. [No. 135, *The Analyst.*]